

Equilibrium and off-equilibrium simulations of chiral-glass order in three-dimensional Heisenberg spin glasses

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Abstract. Spin-glass and chiral-glass orderings in three-dimensional Heisenberg spin glasses are studied both by equilibrium and off-equilibrium Monte Carlo simulations. Fully isotropic model is found to exhibit a finite-temperature chiral-glass transition without the conventional spin-glass order. Although chirality is an Ising-like quantity from symmetry, universality class of the chiral-glass transition appears to be different from that of the standard Ising spin glass. In the off-equilibrium simulation, while the spin autocorrelation exhibits only an interrupted aging, the chirality autocorrelation persists to exhibit a pronounced aging effect reminiscent of the one observed in the mean-field model. Effects of random magnetic anisotropy is also studied by the off-equilibrium simulation, in which asymptotic mixing of the spin and the chirality is observed.

§1. Introduction

Ordering of complex systems has attracted interest of researchers working in the field of numerical simulations. Well-known examples of such complex systems may be a variety of ‘glassy’ systems including window glasses, orientational glasses of molecular crystals, vortex glasses in superconductors and spin-glass magnets. Often, in the dynamics of such complex systems, characteristic slow relaxation is known to occur. It has been a great challenge for researchers to clarify the nature and the origin of these slow dynamics, as well as to get fully equilibrium properties by overcoming the slow relaxation. In particular, *spin glasses* are the most extensively studied typical model system, for which

numerous analytical, numerical and experimental works have been made [1].

Studies on spin glasses now have more than twenty years of history. Main focus of earlier studies was put on obtaining the equilibrium properties of the spin-glass ordering. Owing to extensive experimental studies, it now seems well-established that the spin-glass magnets exhibit an equilibrium phase transition at a finite temperature [1]. From theoretical side, there now seems to be a consensus that the lower critical dimension (LCD) of an Ising spin glass is between $d = 2$ and 3 , while the LCD of vector spin glasses is greater than $d = 3$. In other words, at $d = 3$, only an anisotropic Ising spin glass exhibits an equilibrium spin-glass transition at a finite temperature [2,3], whereas isotropic Heisenberg spin glass exhibits only a zero-temperature transition [4-7]. Meanwhile, it has been known that the magnetic interactions in many of real spin-glass materials are Heisenberg-like, in the sense that the magnetic anisotropy is much weaker than the exchange energy. Apparently, there is a puzzle here: How can one reconcile the absence of the spin-glass order in an isotropic Heisenberg spin glass with the experimental observation? While weak magnetic anisotropy inherent to real materials is often invoked to explain this apparent discrepancy, it remains puzzling that no detectable sign of Heisenberg-to-Ising crossover has been observed in experiments which is usually expected to occur if the observed spin-glass transition is caused by the weak magnetic anisotropy [1,4].

In order to solve this apparent puzzle, a chirality mechanism of experimentally observed spin-glass transitions was recently proposed by one of the authors [6], on the assumption that an isotropic 3D Heisenberg spin glass exhibited a finite-temperature *chiral-glass* transition without the conventional spin-glass order, in which only spin-reflection symmetry was broken with preserving spin-rotation symmetry. ‘Chirality’ is an Ising-like multispin variable representing the sense or the handedness of the noncoplanar spin structures (more detailed definition will be given below). It was argued that, in real spin-glass magnets, the spin and the chirality were “mixed” due to the weak magnetic anisotropy and the chiral-glass transition was then “revealed” via anomaly in experimentally accessible quantities. Meanwhile, theoretical question whether there really occurs such finite-temperature chiral-glass transition in an isotropic 3D Heisenberg spin glass, a crucial assumption of the chirality mechanism, remains somewhat inconclusive [6,7].

More recently, there arose a growing interest both theoretically and experimentally in the *off-equilibrium* dynamical prop-

erties of spin glasses. In particular, aging phenomena observed in many spin glasses [8] have attracted attention of researchers [9,10]. Unlike systems in thermal equilibrium, relaxation of physical quantities depends not only on the observation time t but also on the waiting time t_w , *i.e.*, how long one waits at a given state before the measurements. Recent studies have revealed that the off-equilibrium dynamics in the spin-glass state generally has two characteristic time regimes [9-11]. One is a short-time regime, $t_0 \ll t \ll t_w$ (t_0 is a microscopic time scale), called ‘quasi-equilibrium regime’, and the other is a long-time regime, $t \gg t_w$, called ‘aging regime’ or ‘out-of-equilibrium regime’. In the quasi-equilibrium regime, the relaxation is stationary and the fluctuation-dissipation theorem (FDT) holds. The autocorrelation function at times t_w and $t + t_w$ is expected to behave as

$$C(t_w, t + t_w) \approx q^{\text{EA}} + \frac{C}{t^\lambda} \rightarrow q^{\text{EA}}, \quad (1)$$

where q^{EA} is the equilibrium Edwards-Anderson order parameter. In the aging regime, the relaxation becomes non-stationary, FDT broken, and the autocorrelation function decays to zero as $t \rightarrow \infty$ for fixed t_w .

On theoretical side, both analytical and numerical studies of off-equilibrium dynamics of spin glasses have so far been limited to *Ising-like* models, including the Edwards-Anderson (EA) model with short-range interaction [12-14] or the mean-field models with long-range interaction [11,15-17]. Although these analyses on Ising-like models succeeded in reproducing some of the features of experimental results, it is clearly desirable to study the dynamical properties of *Heisenberg-like* spin-glass models to make a direct link between theory and experiment.

In the present article, we report on our recent results of equilibrium as well as off-equilibrium Monte Carlo simulations on isotropic and anisotropic 3D Heisenberg spin glasses. Ordering properties of both the spin and the chirality will be studied, aimed at testing the validity of the proposed chirality scenario of spin-glass transitions. We note that Monte Carlo simulation is particularly suited to this purpose, since, at the moment, chirality itself is not directly measurable experimentally. By contrast, in numerical simulations, it is quite straightforward to measure the chirality.

§2. Chirality

Frustration in vector spin systems such as the XY and Heisenberg models often causes *noncollinear* or *noncoplanar* spin structures. Such noncollinear or noncoplanar orderings give rise to a nontrivial chirality. In the case of two-component XY spins, one may define a local chirality for the two neighboring spins at the i - and j -th sites by $\kappa_{ij} = \mathbf{S}_i \times \mathbf{S}_j \cdot \mathbf{z} = S_i^x S_j^y - S_i^y S_j^x$. When the spin configuration in the ordered state is noncollinear, the local chirality κ_{ij} defined above takes a nonzero value.

In the case of three-component Heisenberg spins, three spins are necessary to define a scalar chirality. Thus, in the Heisenberg case, one may define a local scalar chirality for three neighboring spins (spin triad) at the i -, j - and k -th sites by $\chi_{ijk} = \mathbf{S}_i \cdot \mathbf{S}_j \times \mathbf{S}_k$. It takes a nonzero value for any noncoplanar spin configurations but vanishes for any planar spin configurations. Note that, in either case, the chirality is a pseudoscalar in the sense that it is invariant under global spin rotation but changes sign under global spin reflection. Possible chiral ordering is related with a breaking of the reflection symmetry with preserving the rotation symmetry.

The model we simulate is the classical Heisenberg model on a simple cubic lattice with the nearest-neighbor random Gaussian couplings, J_{ij} and $D_{ij}^{\mu\nu}$, defined by the Hamiltonian

$$\mathcal{H} = - \sum_{\langle ij \rangle} (J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j + D_{ij}^{\mu\nu} S_i^\mu S_j^\nu), \quad (2)$$

where $\mathbf{S}_i = (S_i^x, S_i^y, S_i^z)$ is a three-component unit vector, and the sum runs over all nearest-neighbor pairs with $N = L \times L \times L$ spins. J_{ij} is the isotropic exchange coupling with zero mean and variance J , while $D_{ij}^{\mu\nu}$ ($\mu, \nu = x, y, z$) is the random magnetic anisotropy with zero mean and variance D which is assumed to be symmetric and traceless, $D_{ij}^{\mu\nu} = D_{ij}^{\nu\mu}$ and $\sum_\mu D_{ij}^{\mu\mu} = 0$.

We define the local chirality at the i -th site and in the μ -th direction, $\chi_{i\mu}$, for three Heisenberg spins by,

$$\chi_{i\mu} = \mathbf{S}_{i+\hat{\mathbf{e}}_\mu} \cdot (\mathbf{S}_i \times \mathbf{S}_{i-\hat{\mathbf{e}}_\mu}), \quad (3)$$

where $\hat{\mathbf{e}}_\mu$ ($\mu = x, y, z$) denotes a unit lattice vector along the μ -axis.

§3. Equilibrium simulations

First, we report on our *equilibrium* Monte Carlo simulation of a fully isotropic 3D Heisenberg spin glass defined by eq.(2) with $D = 0$. Monte Carlo simulation is performed based on an ‘extended ensemble’ method recently developed by Hukushima and Nemoto [18], where the whole configurations at two neighboring temperatures of the same sample are occasionally exchanged with the system remaining at equilibrium. By this method, we succeeded in equilibrating the system down to the temperature considerably lower than those attained in the previous simulations. We run in parallel two independent replicas with the same bond realization and compute an overlap between the chiral variables in the two replicas,

$$q_\chi = \frac{1}{3N} \sum_{i,\mu} \chi_{i\mu}^{\{1\}} \chi_{i\mu}^{\{2\}}. \quad (4)$$

In terms of this chiral overlap, q_χ , the chiral-glass order parameter, $q_{\text{CG}}^{(2)}$, and the Binder cumulant of the chirality, g_{CG} , are calculated by

$$q_{\text{CG}}^{(2)} = [\langle q_\chi^2 \rangle]^2, \quad (5)$$

$$g_{\text{CG}} = \frac{1}{2} \left(3 - \frac{[\langle q_\chi^4 \rangle]}{[\langle q_\chi^2 \rangle]^2} \right), \quad (6)$$

where $\langle \dots \rangle$ represents the thermal average and $[\dots]$ represents the average over bond disorder. At the possible chiral-glass transition point, curves of g_{CG} against T for different L should merge or cross asymptotically for large L .

For the Heisenberg spin, one can introduce an appropriate Binder cumulant in terms of a tensor overlap $q_{\mu\nu}$ ($\mu, \nu = x, y, z$) which has $3^2 = 9$ independent components,

$$q_{\mu\nu} \equiv \frac{1}{N} \sum_i S_{i\mu}^{\{1\}} S_{i\nu}^{\{2\}}, \quad (\mu, \nu = x, y, z), \quad (7)$$

via the relation,

$$g_{\text{SG}} = \frac{1}{2} \left(11 - 9 \frac{\sum_{\mu,\nu,\delta,\rho} [\langle q_{\mu\nu}^2 q_{\delta\rho}^2 \rangle]}{(\sum_{\mu,\nu} [\langle q_{\mu\nu}^2 \rangle])^2} \right). \quad (8)$$

The lattice sizes studied are $L = 6, 8, 10, 12, 16$ with periodic boundary conditions. In the case of $L = 12$, for example, we prepare 50 temperature points distributed in the range $[0.08J, 0.25J]$ for a given sample, and perform 4.7×10^5 exchanges per temperature of the whole lattices combined with the same number of standard single-spin-flip heat-bath sweeps. For $L = 12$, we equilibrate the system down to the temperature $T/J = 0.08J$, which is lower than the minimum temperature attained previously. Sample average is taken over 1500 ($L = 6$), 1200 ($L = 8$), 640 ($L = 10$), 296 ($L = 12$) and 32 ($L = 16$) independent bond realizations. Equilibration is checked by monitoring the stability of the results against at least three-times longer runs for a subset of samples.

The size and temperature dependence of the Binder cumulants of the spin and of the chirality, g_{SG} and g_{CG} , are shown in Fig.1(a) and (b), respectively. As can be seen from Fig.1(a), g_{SG} constantly decreases with increasing L at all temperatures studied, suggesting that the conventional spin-glass order occurs only at zero temperature, consistent with the previous results [1,4-7]. A closer inspection of Fig.1(a), however, reveals that g_{SG} for larger lattices ($L = 10, 12, 16$) exhibits an anomalous “upturn” around $T/J \sim 0.1 - 0.15$, suggesting that a change in the ordering behavior occurs in this temperature range.

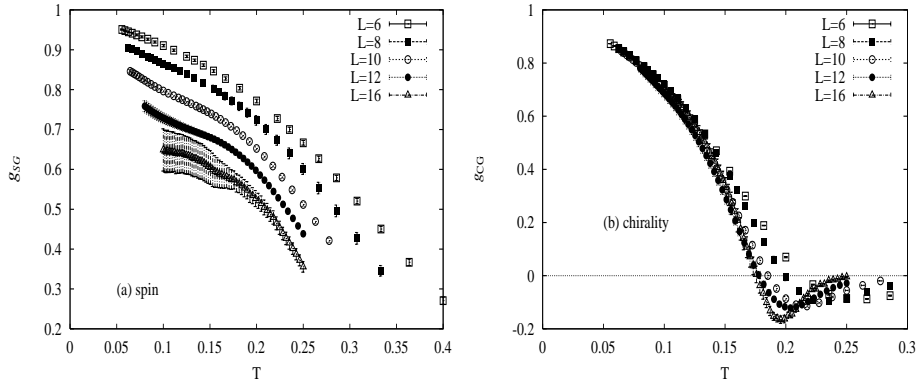


FIG.1 Temperature and size dependence of the Binder cumulants of the spin (a) and of the chirality (b) of a 3D isotropic Heisenberg spin glass.

As can be seen from Fig.1(b), the curves of g_{CG} for different L do not cross, but show a tendency to merge for larger L in the temperature range where the curves of g_{SG} exhibit an anomalous upturn. However, it is not possible to determine only from

the data of g_{CG} whether the system exhibits a finite-temperature transition into the chiral ordered state with some “critical” character, or it exhibits only a zero-temperature transition with rapidly growing correlation length.

Thus, we also have tried the standard finite-size scaling analysis for the chiral-glass order parameter $q_{\text{CG}}^{(2)}$, by adjusting the transition temperature T_{CG} and the exponents β_{CG} and ν_{CG} as fitting parameters. The best fit is obtained for $T_{\text{CG}}/J \sim 0.176$, $\nu_{\text{CG}} \sim 1.15$ and $\beta_{\text{CG}}/\nu_{\text{CG}} \sim 1.30$, with the associated χ_{dof}^2 -value, $\chi_{\text{dof}}^2 \sim 5.3$; see Fig.2(a). If, on the other hand, the same data are fitted to the finite-size scaling form expected for a $T = 0$ transition with nondegenerate ground state, *i.e.*, $T_{\text{CG}} = 0$ and $\beta_{\text{CG}}/\nu_{\text{CG}} = 0$, the best fit is obtained for $\nu_{\text{CG}} \sim 3.2$, but with the associated χ^2 -value, $\chi^2 \sim 29.5$, which is significantly larger than the best χ^2 -value obtained with assuming $T_{\text{CG}} > 0$. Thus, while our individual estimates of T_{CG} , ν_{CG} and $\beta_{\text{CG}}/\nu_{\text{CG}}$ might not be so accurate since we are trying here to determine the three intercorrelated fitting parameters simultaneously, our analysis strongly favors a finite-temperature chiral-glass transition over a zero-temperature transition. Indeed, this conclusion is corroborated with the results of the dynamical simulation to be presented in the next section.

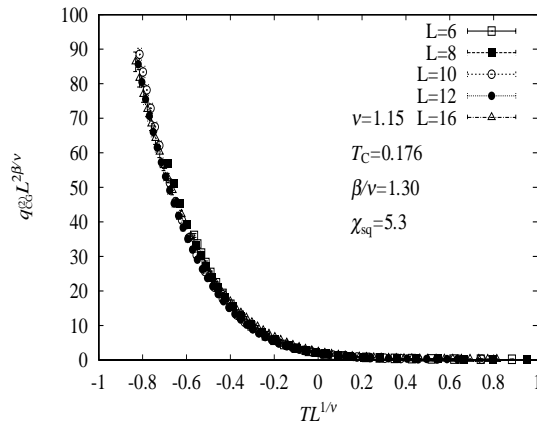


FIG.2 Finite-size scaling plots of the chiral-glass order parameter of a 3D isotropic Heisenberg spin glass, with assuming a finite-temperature transition $T_{\text{CG}} > 0$. The chiral-glass order parameter is divided by the magnitude of the local chirality.[6,7]

In Fig.3, we display the distribution function of the chiral-

overlap defined by

$$P_{\text{CG}}(\bar{q}_\chi) = [\langle \delta(q_\chi - \bar{q}_\chi) \rangle], \quad (9)$$

calculated at a temperature $T/J = 0.1$, below the estimated chiral-glass transition temperature. The shape of the calculated $P_{\text{CG}}(q_\chi)$ is somewhat different from the one observed in the standard Ising-like models such as the 3D EA model or the mean-field SK mode. $P_{\text{CG}}(q_\chi)$ has standard ‘side-peaks’ corresponding to the Edwards-Anderson order parameter $\pm q_{\text{CG}}^{\text{EA}}$, which grow and sharpen with increasing L as is usually the case in the spin-glass state. In addition to the side peaks, a ‘central peak’ at $q_\chi = 0$ shows up for larger L , which also grows and sharpens with increasing L . This latter aspect, *i.e.*, the existence of a central peak growing and sharpening with the system size, is a peculiar feature of the chiral-glass ordered phase never observed in the EA model or the SK model: It might suggest a nontrivial structure in the phase space associated with the chirality. This peculiar feature is reminiscent of the behavior characteristic of some mean-field models showing the so-called *one-step replica-symmetry breaking* (RSB). Thus, it is tempting to deduce that the chiral-glass phase of a 3D Heisenberg spin glass has a character of such one-step RSB. If the ordering is really of such type, no crossing of the Binder ratio needs to occur at $T = T_{\text{CG}}$, and our data of g_{CG} are consistent with the scenario. Further studies are in progress to clarify the situation.

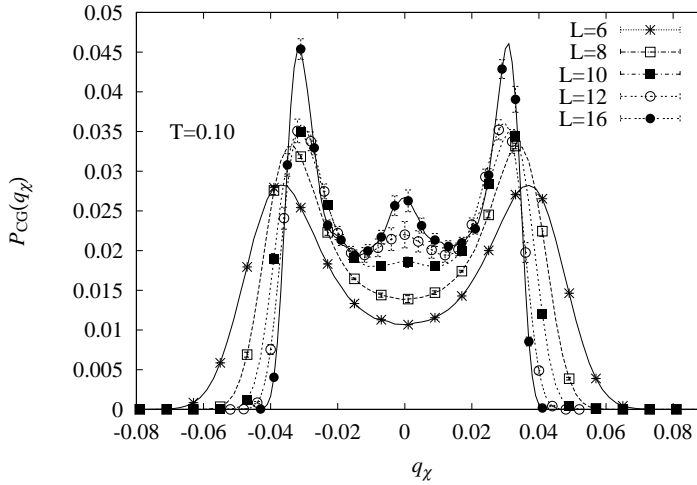


FIG.3 Chiral-overlap distribution function of a 3D isotropic Heisenberg spin glass below T_{CG} . The temperature is $T/J = 0.1$.

§4. Off-equilibrium simulations

In this section, we report on the results of *off-equilibrium* Monte Carlo simulations both on isotropic ($D = 0$) and anisotropic ($D > 0$) models. Unlike the case of equilibrium simulation, the system here is never in full thermal equilibrium. Recent studies have revealed that one can still get many useful information from such off-equilibrium simulations, even including certain *equilibrium* properties. The quantities we are mainly interested here are the spin and chirality autocorrelation functions defined by

$$C_s(t_w, t + t_w) = \frac{1}{N} \sum_i [\langle \vec{S}_i(t_w) \cdot \vec{S}_i(t + t_w) \rangle], \quad (10)$$

$$C_\chi(t_w, t + t_w) = \frac{1}{3N} \sum_{i,\mu} [\langle \chi_{i\mu}(t_w) \chi_{i\mu}(t + t_w) \rangle]. \quad (11)$$

Monte Carlo simulation is performed based on the standard single spin-flip heat-bath method. Starting from completely random initial configurations, the system is quenched to a working temperature. Total of about 3×10^5 Monte Carlo steps per spin [MCS] are generated in each run. Sample average is taken over 30-120 independent bond realizations, four independent runs being made using different spin initial conditions and different sequences of random numbers for each sample. The lattice size mainly studied is $L = 16$ with periodic boundary conditions, while in some cases lattices with $L = 12$ and 24 are also studied.

Let us begin with the fully isotropic case, $D = 0$. The spin and chirality autocorrelation functions at a low temperature $T/J = 0.05$ are shown in Fig.4 as a function of t . For larger t_w , the curves of the spin autocorrelation function C_s come on top of each other in the long-time regime, indicating that the stationary relaxation is recovered and aging is interrupted. This behavior has been expected because the 3D Heisenberg spin glass has no standard spin-glass order [1,4-7]. Similar interrupted aging was observed in the 2D Ising spin glass which did not have an equilibrium spin-glass order [13]. By contrast, the chiral autocorrelation function C_χ shows an entirely different behavior: Following the initial decay, it exhibits a clear plateau around $t \sim t_w$ and then drops sharply for $t > t_w$. It also shows an eminent aging effect, namely, as one waits longer, the relaxation becomes slower and the plateau-like behavior at $t \sim t_w$ becomes more pronounced.

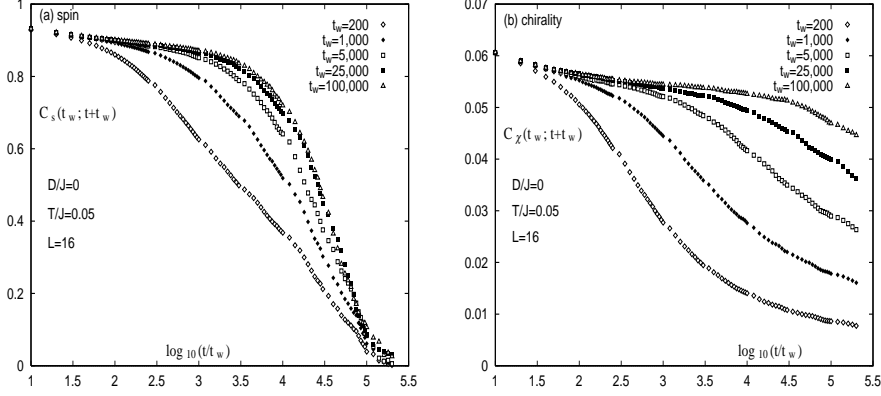


FIG.4 Spin (a) and chirality (b) autocorrelation functions of a 3D isotropic Heisenberg spin glass at a temperature $T/J = 0.05$ plotted versus $\log_{10} t$ for various waiting times t_w . The lattice size is $L = 16$ averaged over 66 samples.

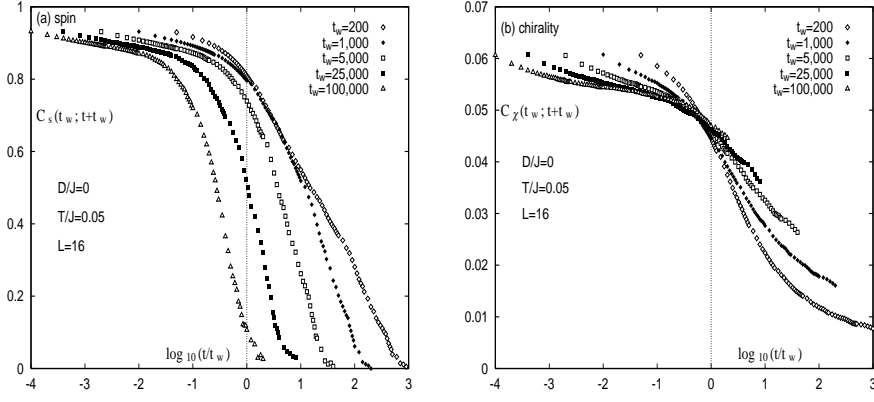


FIG.5 The same data as in Fig.4, but plotted versus $\log_{10}(t/t_w)$.

In Fig.5, C_s and C_χ are replotted as a function of the scaled time t/t_w . Reflecting its interrupted aging, the curves of C_s for larger t_w now lie below the ones for smaller t_w (*subaging*). By contrast, the curves of C_χ for various t_w cross around $t/t_w \sim 1$, and at $t > t_w$, the data for larger t_w lie *above* the ones for smaller t_w (*superaging*). Such superaging behavior of C_χ means that the aging in chirality is more enhanced than the one expected from the naive t/t_w -scaling. Note that, although the chirality is an Ising-like variable from symmetry, the observed superaging behavior is in contrast to the aging behavior of the 3D EA Ising model which was found to satisfy a good t/t_w -scaling in the aging regime [13]. It should also be noticed that the plateau-like behavior observed

here has been hardly noticeable in simulations of the 3D EA Ising model. Rather, the behavior of C_χ observed here is reminiscent of the one observed in the mean-field model such as the SK model [15-17]. This correspondence might suggest that an effective interaction between the chiralities is long-ranged. Indeed, at least in case of XY spins, the interaction between chiralities is known to be Coulombic [19], while such analytical information is not available for Heisenberg spins.

While the plateau-like behavior observed in C_χ is already suggestive of a nonzero *chiral* Edwards-Anderson order parameter, $q_{CG}^{EA} > 0$, more quantitative analysis similar to the one recently done by Parisi *et al* for the 4D Ising spin glass [14] is performed to extract q_{CG}^{EA} from the data of C_χ in the quasi-equilibrium regime. Finiteness of q_{CG}^{EA} is also visible in a log-log plot of C_χ versus t as shown in the inset of Fig.6, where the data show a clear upward curvature. We extract q_{CG}^{EA} by fitting the data of C_χ for $t_w = 3 \times 10^5$ to the power-law form of eq.(1) in the time range $40 \leq t \leq 3,000$ satisfying $t/t_w \leq 0.01$. Stability of the result has been checked by examining the robustness of the result against the change in the value of t_w and the time window used in the fit. The obtained q_{CG}^{EA} , plotted as a function of temperature in Fig.6, clearly indicates the occurrence of a finite-temperature chiral-glass transition at $T_{CG}/J = 0.157 \pm 0.01$ with the associated order-parameter exponent $\beta_{CG} = 1.1 \pm 0.1$. The size dependence turns out to be rather small, although the mean values of q_{CG}^{EA} tend to slightly increase around T_{CG} with increasing L . Since both finite-size effect and finite- t_w effect tend to underestimate q_{CG}^{EA} , one may regard the present result as a rather strong evidence of the occurrence of a finite-temperature chiral-glass transition.

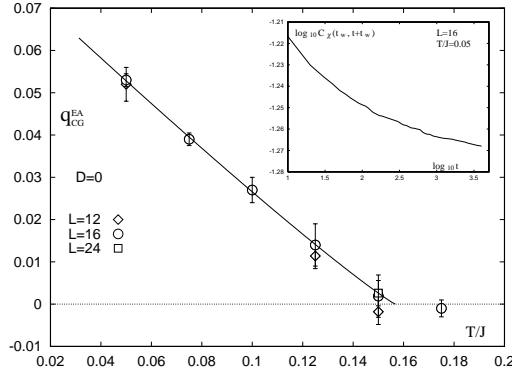


FIG.6 Temperature dependence of the Edwards-Anderson order pa-

parameter of the chirality of a 3D isotropic Heisenberg spin glass. The data are averaged over 30-120 samples. Inset exhibits the log-log plot of the t -dependence of the chirality autocorrelation function in the quasi-equilibrium regime for $L = 16$ and $t_w = 3 \times 10^5$.

If one compares the present estimate with the static result in the preceding section, the estimated transition temperature is a bit lower than, but is roughly consistent with the static estimate $T_{CG}/J \sim 0.17$, while the exponent β_{CG} is somewhat smaller than the static estimate, $\beta_{CG} \sim 1.5$. In any case, the obtained exponent $\beta_{CG} \sim 1.1$ (or still larger value from the static estimate) is considerably larger than the value of the 3D EA model $\beta \sim 0.5$ [1-3], and is rather close to the value of the mean-field model $\beta = 1$. This suggests that the universality class of the chiral-glass transition of the 3D Heisenberg spin glass might be different from that of the standard 3D Ising spin glass. According to the chirality mechanism, the criticality of real spin-glass transitions should be the same as that of the chiral-glass transition of an isotropic Heisenberg spin glass, so long as the magnitude of random anisotropy is not too strong. If one tentatively accepts this scenario, the present result opens up a new interesting possibility that the universality class of many of real spin-glass transitions might differ from that of the standard Ising spin glass, contrary to common belief.

In the presence of weak anisotropy $D > 0$, chirality scenario predicts at the static level that the transition behavior of chirality remains essentially the same as in the isotropic case, whereas the spin is mixed into the chirality, asymptotically showing the same transition behavior as the chirality [6]. In order to see whether such “spin-chirality mixing” occurs in the off-equilibrium dynamics, further dynamical simulations are performed for the models with random anisotropies $D/J = 0.01 \sim 1$. While chirality exhibits essentially the same dynamical behavior as in the isotropic case (not shown here), the behavior of spin at $t > t_w$ changed significantly in the presence of anisotropy. As an example, the spin autocorrelation in the case of weak anisotropy $D/J = 0.01$ is shown in Fig.7. Even for such small anisotropy, spin is found to show *superaging* behavior asymptotically at $t \gg t_w$ similar to that of the chirality in *zero* and weak anisotropies, demonstrating the spin-chirality mixing.

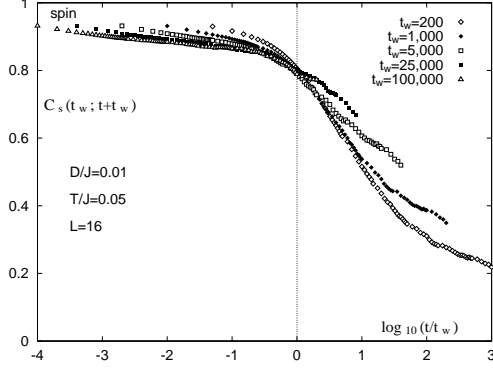


FIG.7 Spin autocorrelation function of the weakly anisotropic 3D Heisenberg spin glass with $D/J = 0.01$ plotted versus $\log_{10}(t/t_w)$. The lattice size is $L = 16$ averaged over 60 samples and the temperature is $T/J = 0.05$.

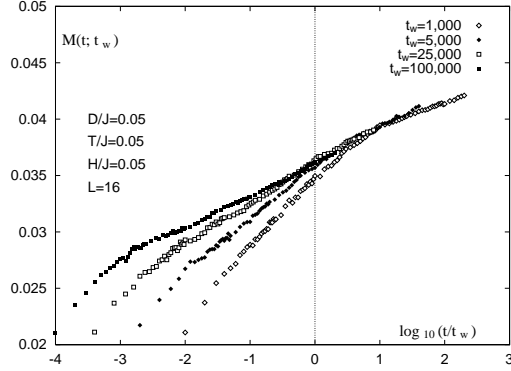


FIG.8 Zero-field-cooled magnetization of an anisotropic 3D Heisenberg spin glass with $D/J = 0.05$ plotted versus $\log_{10} t$. The field is $h/J = 0.05$ and the temperature is $T/J = 0.05$. The lattice size is $L = 16$ averaged over 80 samples.

Experimentally, thermoremanent magnetization (TRM) or zero-field-cooled (ZFC) magnetization is found to show an approximate t/t_w -scaling in the aging regime, with small deviation from the perfect scaling in the direction of subaging [9]. Although this seems in apparent contrast to the present result, it should be noticed that standard aging experiments have been made by measuring the magnetic response, not the autocorrelation. Recent numerical simulation by Yoshino *et al* revealed that, at least in the case of the SK model, TRM showed the subaging even when the spin correlation showed the superaging [20]. Thus, we also calculate the ZFC magnetization for an anisotropic model

with $D/J = 0.05$: After the initial quench, the system is evolved in zero field during t_w MCS. Then, an external field of intensity $H/J = 0.05$ is turned on and the subsequent growth of the magnetization $M(t; t_w)$ is recorded. As can be seen from Fig.8, the data show the near t/t_w -scaling in the aging regime $t > t_w$ where the spin-autocorrelation shows the superaging. Thus, the observed tendency is roughly consistent with experiments. It might be interesting to experimentally investigate the aging properties of *spin correlations* of Heisenberg-like magnets in search for possible superaging behavior.

§5. Summary

In summary, spin-glass and chiral-glass orderings in 3D Heisenberg spin glasses are studied with and without random anisotropy by Monte Carlo simulations. The results are basically consistent with the chirality mechanism: In the isotropic case, clear evidence of the occurrence of a finite-temperature chiral-glass transition without the conventional spin-glass order is presented both by equilibrium and off-equilibrium simulations. Spin and chirality show very different dynamical behaviors consistent with the ‘spin-chirality separation’. While the spin autocorrelation exhibits only an interrupted aging, the chirality autocorrelation persists to exhibit a pronounced aging effect reminiscent of the one observed in the mean-field model. The universality class of the chiral-glass transition appears to be different from that of the standard Ising spin glass. In the anisotropic case, the off-equilibrium simulation indicates that the spin shows the same asymptotic behavior as the chirality in the isotropic case, demonstrating the ‘spin-chirality mixing’ due to magnetic anisotropy.

The authors are thankful to I. Campbell, R. Orbach, H. Takayama, E. Vincent, A.P. Young, L.F. Cugliandolo, M. Ocio, H. Rieger, K. Nemoto and H. Yoshino for useful discussion. The numerical calculation was performed on the FACOM VPP500 at the supercomputer center, ISSP, University of Tokyo and on the HITACHI SR-2201 at the supercomputer center, University of Tokyo.

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